Semiclassical optics as an alternative to nonlocality

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Abstract. We show that the Wigner formalism in quantum optics is capable of interpretation as a classical wave field with the addition of a zeropoint contribution. We assume that only those states whose Wigner function is positive are real states, and show that this is not a serious restriction. Phenomena currently classified as nonlocal in the standard description of quantum optics may then be understood as arising from correlations between stochastic Maxwell fields. In particular we establish that "entanglement" between pairs of photons with a common origin, in an atomic cascade or a nonlinear crystal, occurs because the two light signals have amplitudes and phases, both below and above the zeropoint intensity level, which are correlated with each other. An essential feature of the Wigner reformulation is that the normal-ordering theory of detection, which has its origin in the collapse of the wave function, must be replaced by a theory which, with reasonable efficiency, separates a signal from the zeropoint background. That requires a resolving time window of an appropriate length. The theory explains, in a local manner, the violation of homogeneous Bell inequalities in all optical tests which have been performed. We predict a new phenomenon in nonlinear crystals, namely Spontaneous Parametric Up Conversion; it has been given no explanation within conventional quantum optics.

1 Introduction

At the 1927 Solvay conference Einstein entered his objection to the new Quantum Mechanics (QM). That objection was to what was known as "collapse of the wave packet", and it was subsequently clarified in two articles[1, 2]. He showed that QM is *nonlocal*, and therefore contrary not only to his Relativity Theory, but also to all hitherto accepted norms of scientific explanation.

Bohm[3] simplified and sharpened the thought experiment of Einstein, Podolsky and Rosen[1] (EPR). Considering this version of EPR, Bell[4] showed that correlations between the internal variables of a pair of separated spin-1/2 particles with a common origin, according to any local theory, must satisfy a certain inequality, which we shall call an Inhomogeneous Bell Inequality (IBI), and that there are circumstances, according to QM, in which an IBI is violated.

The possibility of observing such violations gave rise to a great deal of experimental activity. Curiously, the only useful experiments have been made[5, 6, 7, 8, 9, 10] with pairs of light signals[1], the light photons being interpreted as EPR particles, and the internal variables being their polarizations. Photons are considered to have spin one, but the absence of a longitudinal spin component means that there are only two independent photon modes of given momentum, so observing their polarization is considered to be like observing a spin component of a spin-1/2 particle.

One would expect considerable care to be exercised in analyzing such experiments, since the corpuscularity of photons is very problematic. This was emphasized by Max Planck[12], widely considered as the creator of Quantum Theory, as early as 1907, but also by his then young opponent Albert Einstein, who in 1951 remarked[13] "Nowadays every Tom, Dick and Harry thinks he knows what a photon is, but he is mistaken". More recently Willis Lamb[14] has said people should be required to have a licence before being allowed to speak of photons. Clearly Lamb did not have the editors of *Physical Review Letters* in mind as the appropriate licensing authority, because, since 1982, virtually all the articles on Bell experiments, published there and in other authoritative journals, have simply assumed that photodetectors are devices which treat all incident photons on an equal basis. Such a "fair sampling" assumption is now so standard that it is not acknowledged at all any more, yet its imposition effectively limits the range of local theories to those which accept the corpuscularity of the light field. That means, in particular, that Max Planck's own description, which we shall review in this article, and which we shall show to be consistent with locality, is ruled out of consideration.

Before 1982 the intellectual climate was more enlightened. In early designs of their experiment of 1973, Clauser and Freedman used Fair Sampling in the form of the Clauser-Horne-Shimony-Holt (CHSH)[15] hypothesis. However, shortly afterwards Clauser and Horne (CH)[16] made the impor-

[1]Proposals have been made for experiments with atoms[11], but they have not so far been realised. Some attempts to test Bell inequalities have been made with gamma rays from positronium decay, and with protons[6]. In all of these cases the inability to make unambiguous spin measurements has made a true test of IBI impossible.

tant distinction between inhomogeneous (IBI) and homogeneous (HBI) Bell inequalities. Testing an IBI requires that we compare certain coincidence rates in two separated detectors with the singles rates of the two detectors. Nobody needed to actually perform such an experiment, because singles rates with all detectors, before 1974 and also since 1974, are at least ten times all the coincidence rates. So, taking into account this low detector efficiency, the QM prediction actually satisfies the IBI. To arrive at an experimental design in which the QM prediction violates IBI we require detectors whose efficiency exceeds 67%. The authors have been repeatedly assured, since 1982, that such a technological advance is just around the corner, but a recent article[17] by one of the principal participants in the Aspect[7, 8] series of experiments seems to indicate, twenty years on, that the effort to achieve it has now been abandoned.

Clauser and Horne recognized that, because of the limitations of light detectors, an experimental test between quantum optics and its putative local competitors was possible only so long as these competitors accepted some restriction on the range of local theories to be considered. They introduced the No Enhancement Hypothesis (NEH), which is that a given light signal, originating in an atomic cascade for example, has a certain probability of activating a detector, and that, if a polarizer is interposed between the cascade and the detector, that detection probability cannot increase. Given NEH, we may derive an HBI, between coincidence rates with polarizers in place and coincidence rates without one or both polarizers, and this was made the basis of the experimental procedure, used by Clauser and Freedman.

In the latter experiment the HBI was found to be violated. To us the conclusion to be drawn is obvious; NEH plus locality cannot be true, so, since locality (or, what is the same thing, causality) is the basis of all science, NEH is not true. So the opposite of NEH is true, that is enhancement is a *new phenomenon* discovered by Clauser and Freedman. Let us be explicit.

> In the total set of signals from an atomic cascade there is a subset whose detection probability increases as a result of passing through a linear polarizer.

Stochastic optics[18, 19] is a form of semiclassical radiation theory. As in QM, there is a wave field $[\mathbf{E}(\mathbf{x}, t)\,, \mathbf{B}(\mathbf{x}, t)]$. It satisfies the Maxwell propagation equation, which is the Schrödinger equation for this particular field; so first quantization of the light field goes back to the 1870s! In contrast with the Schrödinger equation for massive particles, there is no problem in treating the Maxwell field as a real object; it propagates, in all media, at velocity not greater than c. We claim that second quantization of the light field is equivalent to taking account of the fluctuations of that field, that is recognizing its stochastic nature. Whether light detectors are activated, or not, in an interval $0 \leq t \leq \tau$, depends on the value of a certain quadratic functional of $\mathbf{E}$, involving an integration from 0 to τ. We shall say more about the precise form of the quadratic functional in the following sections. Essentially it is a time integral of the intensity within an appropriate frequency range.

The stochastic element was introduced into modern optics by Brown and Twiss[20]. They showed that a thermal source can be represented as a gaussian stochastic process, and that a photodetector, operated with a very long ($\sim 5 \times 10^3$s) detection window, converts the light field into a current field. Then the correlation between the currents in two separated detectors may be obtained as a fourth-order correlation of the incident field, which, from the assumed gaussian property of the latter, reduces to a product of two field correlations. This technique forms the basis of a method for measuring stellar diameters.

The analysis is not always so simple. There is a category of light field, which is nowadays called *nonclassical*, exhibiting new phenomena such as antibunching[21] and anticorrelation[22, 23]. We have shown that the key to a local explanation of these phenomena also explains the enhancement phenomenon, namely the zeropoint field (ZPF), introduced into radiation theory by Max Planck[24] in 1911, and revived in more recent times under the title of Stochastic Electrodynamics[25, 26, 27] (SED). Originally SED was an attempt to explain atomic structure as an interaction of point electrons with the ZPF. Though we were participants in this ambitious programme, we consider that it has failed. Nevertheless, a semiclassical version of it, in which quantized atoms interact with a classical Maxwell field which includes "vacuum" fluctuations, survives, and that is what we shall describe in the following sections. The notion of real "vacuum" fluctuations is also a feature of some presentations of Quantum Electrodynamics (QED)[28], and has been used to explain such phenomena as spontaneous emission, the Casimir effect and the greater (that is, nonrelativistic) part of the Lamb shift.

A correlated "nonclassical" light field is generated when two modes of the field are coupled, so that their amplitudes cease to be independent. This may occur as a result of their interaction with a nonlinear polarizable medium, when the latter is pumped by a laser. Examples of such a medium are an atom which has been pumped into an excited state, from which it decays, either by resonance fluorescence or by a three-level cascade process, and a certain category of optically active crystal. The input field, for any of these systems, may be modelled, as we shall show in the following sections, by the (idealized) purely sinusoidal laser plus the (gaussian) zeropoint field; we shall show that the output then exhibits all the "nonclassical" features, including enhancement, that we mentioned above.

A novel feature of stochastic optics[18, 19] (SO), which is the term we introduced for the semiclassical version of SED, is that the detection probability for any mode of the field is not linear in that mode's intensity. Because there is a considerable amount of energy in the ZPF, and because the *dark rate*, that is the rate at which the detector fires in the absence of a signal, is very small, there must be a threshold intensity, corresponding to the ZPF intensity, below which the detector does not fire. Such a subtraction is made formally in standard quantum optics, by what is called the *normal ordering* procedure, but what is required in order to complete the SO programme is a theory of detection which treats dark-rate events and signal detections on an equal footing. Note that the nonlinearity of the detection process already means that we have gone beyond the constraints on classical fields imposed by, for example, Clauser[22] and Grangier, Roger and Aspect[23] in their discussions of the anticorrelation phenomenon.

The formulation of a detection theory which covers all the situations we have listed is a formidable problem, and we have not yet been able to complete it. Such a theory will have to contain, as adjustable parameters, the threshold of the detector, its resolving time window, its range of frequency filtering, and the lens system used to focus the signal on to the detecting material. Such progress as we have made is mainly for detectors of long resolving window; this may be taken to include the human eye, whose resolving time is around 0.2s. Thus our theory of detection is able to explain[29] the visibility of stars in the sky, against a very large ZPF background. A phenomenon which may be observed with a long resolving window, and which has not been predicted by conventional quantum optics, is Spontaneous Parametric Up Conversion (SPUC), to be described in Section 4 below.

2 Wigner representation of the light field

The states which form the basis for the quantum optical description of the light field are the *Fock states*, which are generated by applying the *creation operators* $\hat{a}^{\dagger}_{\mathbf{k},\lambda}$ to the hilbert-space vector

$$|0\rangle = \prod_{\mathbf{k},\lambda} |0_{\mathbf{k},\lambda}\rangle \ , \tag{1}$$

which represents the vacuum. The whole state space is then spanned by the set of vectors

$$|\{n_{\mathbf{k},\lambda}\}\rangle = \prod_{\mathbf{k},\lambda} |n_{\mathbf{k},\lambda}\rangle = \prod_{\mathbf{k},\lambda} (n_{\mathbf{k},\lambda}!)^{-1/2} (\hat{a}^{\dagger}_{\mathbf{k},\lambda})^{n_{\mathbf{k},\lambda}} |0_{\mathbf{k},\lambda}\rangle \ , \tag{2}$$

which represents a state having $n_{\mathbf{k},\lambda}$ *photons* of wave number $\mathbf{k}$ and polarization λ. The latter index takes either the value 1 or 2. The most general pure state is a superposition of these, that is

$$|\Phi\rangle = \sum \phi(\{n_{\mathbf{k},\lambda}\}) |\{n_{\mathbf{k},\lambda}\}\rangle \ , \ \sum |\phi|^2 = 1 \ . \tag{3}$$

The Wigner function of this state is defined as

$$W_{\Phi}(\alpha) = W_{\Phi}(\{\alpha_{\mathbf{k},\lambda}\}) = \langle\Phi|\hat{W}(\{\alpha_{\mathbf{k},\lambda}\})|\Phi\rangle \ , \tag{4}$$

where

$$\hat{W}(\{\alpha_{\mathbf{k},\lambda}\}) = \prod_{\mathbf{k},\lambda} \frac{1}{\pi^2} \int e^{\beta_{\mathbf{k},\lambda}(\hat{a}^{\dagger}_{\mathbf{k},\lambda} - \alpha^{*}_{\mathbf{k},\lambda}) - \beta^{*}_{\mathbf{k},\lambda}(\hat{a}_{\mathbf{k},\lambda} - \alpha_{\mathbf{k},\lambda})} d^2\beta_{\mathbf{k},\lambda} \ . \tag{5}$$

For instance, the Wigner function of the vacuum is

$$W_{\text{vac}}(\alpha) = \prod_{\mathbf{k},\lambda} (2/\pi) e^{-2|\alpha_{\mathbf{k},\lambda}|^2}. \tag{6}$$

The full set of quantum states is obtained by extending the set $|\Phi\rangle$ to *mixtures* of the form

$$\rho = \sum |\Phi\rangle P_{\Phi} \langle\Phi| \ , \ 0 \leq P_{\Phi} \leq 1 \ , \ \sum P_{\Phi} = 1 \ . \tag{7}$$

The Wigner function in nonrelativistic QM[30] plays the role of a pseudo-probability distribution; its marginals with respect to position and momentum separately give the quantum probabilities for each of these variables, but the function itself is not positive definite. There are great difficulties in interpreting the Wigner function as a true probability distribution in QM. Nevertheless we propose that for the light field, which satisfies the linear equations of Maxwell, the Wigner function is positive for all physically realizable states, and therefore provides a classical stochastic interpretation of quantum optics.

2.1 Some simple states

In this subsection we consider the idealized situation where only a single mode of the field contains photons, so that the set $\{n_{\mathbf{k},\lambda}\}$ contains only a single nonzero member, and the number states are designated simply $|0\rangle, |1\rangle, |2\rangle \ldots$. An important state is the *coherent*, which is an idealized form of the continuous-wave laser, namely

$$|\alpha'\rangle = \mathrm{e}^{\alpha'\hat{a}^\dagger - \alpha'^*\hat{a}}|0\rangle = \sum_n \frac{\alpha'^n}{\sqrt{n!}}\,\mathrm{e}^{-|\alpha'^2|/2}|n\rangle \ . \tag{8}$$

Its Wigner function is

$$W_{\alpha'}(\alpha) = (2/\pi)\mathrm{e}^{-2|\alpha-\alpha'|^2} \ . \tag{9}$$

A squeezed version of this state is given by

$$|\alpha', s\rangle = e^{s(\hat{a}^{\dagger 2} - \hat{a}^2)}|\alpha'\rangle \ , \tag{10}$$

where s is taken as real, and its Wigner function is, putting $\alpha = \beta + i\gamma, \alpha' = \beta' + i\gamma'$,

$$W_{\alpha',s} = (2/\pi)\exp[-2e^{2s}(\beta-\beta')^2 - 2e^{-2s}(\gamma-\gamma')^2] \tag{11}$$

Note that both of these states have positive Wigner functions. That is not the case with the number states. For example the one-photon state $|1\rangle$ has the Wigner function

$$W_1(\alpha) = (4|\alpha|^2 - 1)W_0(\alpha) \ , \tag{12}$$

where $W_0(\alpha)$ is the vacuum Wigner function for a single mode, that is

$$W_0(\alpha) = (2/\pi)e^{-2|\alpha|^2} \ . \tag{13}$$

In all of the above examples, the full Wigner function is given by the product of the single-mode function with $W_0(\alpha_{\mathbf{k},\lambda})$ for each of the "unoccupied" modes.

2.2 What is a classical state?

At the basis of our semiclassical description is the proposal that $W_0(\alpha_{\mathbf{k},\lambda})$ is a real distribution of amplitudes $\alpha_{\mathbf{k},\lambda}$. Indeed it is precisely the distribution proposed by Max Planck in his original formulation of the ZPF, whereby the electric field in a cube of side L is represented as a *stochastic field*

$$\begin{aligned} \mathbf{E}(\mathbf{r},t) &= \mathbf{E}^{(+)}(\mathbf{r},t) + \mathbf{E}^{(-)}(\mathbf{r},t) \ , \\ \mathbf{E}^{(-)}(\mathbf{r},t) &= [\mathbf{E}^{(+)}(\mathbf{r},t)]^* \ , \\ \mathbf{E}^{(+)}(\mathbf{r},t) &= \sum_{\mathbf{k},\lambda} \sqrt{\frac{\hbar|\mathbf{k}|}{2\epsilon_0 L^3}}\ \alpha_{\mathbf{k},\lambda}\mathbf{u}_{\mathbf{k},\lambda}e^{i\mathbf{k}.\mathbf{r}-|\mathbf{k}|ct} \ , \end{aligned} \tag{14}$$

where $\mathbf{u}_{\mathbf{k},\lambda}$ is a unit vector giving the direction of polarization. All of the random variables $\alpha_{\mathbf{k},\lambda}$ have nonzero variance; that is why we put "unoccupied" in quotation marks; the absence of photons does not mean the absence of radiation!

The expression for $\mathbf{E}$ as a stochastic field is formally almost identical with that used in quantum optics, where the corresponding quantity $\hat{\mathbf{E}} = \hat{\mathbf{E}}^{(+)} + \hat{\mathbf{E}}^{(-)}$, with $\hat{\mathbf{E}}^{(-)} = [\hat{\mathbf{E}}^{(+)}]^\dagger$, is an operator, and the random mode amplitudes are replaced by creation operators, that is

$$\hat{\mathbf{E}}^{(+)}(\mathbf{r},t) = \sum_{\mathbf{k},\lambda} \sqrt{\frac{\hbar|\mathbf{k}|}{2\epsilon_0 L^3}}\ \hat{a}_{\mathbf{k},\lambda}\mathbf{u}_{\mathbf{k},\lambda}e^{i\mathbf{k}.\mathbf{r}-i|\mathbf{k}|ct} \ . \tag{15}$$

In order to discuss the classical nature, or otherwise, of various quantum states, we shall introduce a new category. We follow the convention of quantum optics in calling "classical" those states which have a nonsingular Glauber P-representation

$$\rho = \int |\alpha'\rangle P(\alpha')\langle\alpha'|\, d^2\alpha' \ , \tag{16}$$

where $|\alpha'\rangle$ is a coherent state. (We are keeping within the single-mode set for the moment). The Wigner function of this state is

$$W_\rho(\alpha) = (2/\pi) \int P(\alpha') e^{-2|\alpha-\alpha'|^2} \, d^2\alpha' \; . \tag{17}$$

Since this is positive, it follows that all classical states have positive Wigner functions. We insist that the ZPF oscillations are real, so we need a name for those states having positive Wigner functions. We propose to call them "real", because all phenomena involving such states may be interpreted in a local realist manner, as we shall show in subsequent sections. So, in our understanding, the squeezed state described in the previous subsection is real. The singular character of the P-representation for this state arises because the variances of certain ZPF amplitudes have been depressed below their normal free-space value. The classification of states is summarized[31] in Fig.1, where we depict the set of classical states by a small circle, contained within the wider, real set. The latter has an overlap with the quantum states, which we designate as laboratory (or LAB) states. There are quantum states, like all of the number states, which lie outside even our real set, and we have placed the representative point for a single-photon state at A in the figure. As is shown by eq.(13), this state lies outside the real set, but the squeezed state lies within it.

A second class of nonclassical but real states are those which exhibit antibunching, that is a sub-Poissonian pattern of counts in a photodetector

$$\langle n^2 \rangle - \langle n \rangle^2 = \alpha \langle n \rangle \; , \tag{18}$$

with $\alpha < 1$. These states were discussed in Ref.[18]; there is no difficulty in explaining their counting statistics, on the basis of a source whose intensity shows a regular oscillatory behaviour, as is observed, for example in resonance fluorescence, as studied by Kimble and Mandel[21]. So this set of states is also real.

Now we are in a position to show what characterizes, physically rather than formally, the classical states[32]. They are states for which there is a signal, above the ZPF, which is independent of the ZPF. In contrast, non-classical real states are those for which a separation into signal and ZPF is not possible. A state is classical if the total field $\mathbf{E}(\mathbf{r},t)$ can be decomposed into two independent parts, $\mathbf{E_0}(\mathbf{r},t)$ and $\mathbf{E_1}(\mathbf{r},t)$ representing ZPF and signal respectively. When this is the case, all optical phenomena are associated with the signal alone, and the ZPF may be ignored altogether, as is the situation in classical optics. The independence of the fields implies an independence of the corresponding amplitudes, $\{\alpha^0_{\mathbf{k},\lambda}\}$ and $\{\alpha^1_{\mathbf{k},\lambda}\}$ in expansions like (14). If the probability densities of these are $W_0(\{\alpha^0_{\mathbf{k},\lambda}\})$ and $W_1(\{\alpha^1_{\mathbf{k},\lambda}\})$, then the density of the total field will be their convolution, that is

$$W(\{\alpha\}) = \prod \int W_1(\{\alpha'\}) W_0(\{\alpha - \alpha'\}) d^{2N}\alpha' \ , \tag{19}$$

which is identical with (17) if we identify $W_1(\alpha)$ with $P(\alpha)$. So, to summarize, the existence of a positive P-function is a necessary and sufficient condition for being able to decompose the field into independent signal and ZPF parts.

We now turn to a central problem of our programme, namely the Wigner description of number states like A. Taking into account the complete set of modes, including the "unoccupied" ones, the Wigner function eq.(13) may be written as

$$\mathcal{W}_1(\alpha) = (4|\alpha_{\mathbf{k},\lambda}|^2 - 1) \ \mathcal{W}_0(\alpha) \ . \tag{20}$$

Of course this is not always positive. However, it is necessary to bear in mind that the experimental situations[22, 23], in which something like a one-photon state has been reported, must necessarily involve the observations of wave packets rather than single-mode signals. Indeed, the latter, which fill the whole of space and time, are not at all physical objects. A wave packet has the hilbert-space representation

$$|\xi(\mathbf{x})\rangle = \sum_{\mathbf{k},\lambda} \xi_{\mathbf{k},\lambda} e^{i\mathbf{k}.\mathbf{x}} a^{\dagger}_{\mathbf{k},\lambda} |0\rangle \ , \tag{21}$$

where $\{\xi_{\mathbf{k},\lambda}\}$ are a set of random, but not independent variables satisfying

$$\sum_{\mathbf{k},\lambda} |\xi_{\mathbf{k},\lambda}|^2 = 1 \ , \tag{22}$$

and which, furthermore, are nonzero only for a set of vectors $\mathbf{k}$ falling within a small ellipsoidal region centred at $\mathbf{k}'$. It is also necessary to bear in mind that there is no way of controlling the moment at which such a packet is emitted, in the atomic-cascade situation used for such experiments, and this may be taken into account by forming an appropriate mixture[32, 33] of such wave-packet states. We have been able to show that, for any one-photon state, like the one designated as A in Fig.1, a corresponding mixed state, designated B, may be constructed. The latter state lies precisely on the boundary of the (Wigner) classical set, and the size of the ellipsoidal region is consistent with the experimental variation of **k**.

3 A realist theory of detection

3.1 Quantum theory of detection

In order to complete the Wigner function approach to quantum optical experiments we must give an expression for the counting rate in terms of the radiation arriving at a detector. The hilbert-space, or photon, formalism is based on normal ordering, that is putting creation operators to the left and annihilation operators to the right. We begin with the idealized case where the radiation field is represented by a single mode. Then, working in the Heisenberg picture, the singles detection probability per unit time is

$$\begin{aligned} p^q &\propto \langle\Phi|\hat{b}^\dagger_{\mathbf{k}}(t)\hat{b}_{\mathbf{k}}(t)|\Phi\rangle \\ &= \tfrac{1}{2}\langle\Phi|\left[\hat{b}^\dagger_{\mathbf{k}}(t)\hat{b}_{\mathbf{k}}(t)+\hat{b}_{\mathbf{k}}(t)\hat{b}^\dagger_{\mathbf{k}}(t)-1\right]|\Phi\rangle \\ &= \int W(\{\alpha_{\mathbf{k}}\})\left[|\beta_{\mathbf{k}}(t;\alpha_{\mathbf{k}'})|^2-\tfrac{1}{2}\right]d^2\alpha_{\mathbf{k}'} \\ &\equiv \langle|\beta_{\mathbf{k}}|^2-\tfrac{1}{2}\rangle_W \end{aligned} \tag{23}$$

where $|\ \Phi\rangle$ is the initial state of the radiation, $W(\alpha_{\mathbf{k}'},\alpha^*_{\mathbf{k}'})$ the corresponding Wigner function, and $\hat{b}_{\mathbf{k}}(t)$ ($\hat{b}^\dagger_{\mathbf{k}}(t)$) the time-dependent creation (annihilation) operator, $\beta_{\mathbf{k}}$ and $\beta^*_{\mathbf{k}}$ being the corresponding amplitudes in the Wigner formalism. The first equality derives from the use of the commutation relations and the second is the passage to the Wigner representation. In the latter expression we have exhibited the dependence of the amplitude $\beta_{\mathbf{k}}$ on time and on the initial amplitudes, $\alpha_{\mathbf{k}'}$,. The symbol $\langle\rangle_W$ means the average weighted with the Wigner function. In what follows we shall omit the subindex W.

In the general case of many modes, from (14) it is straightforward to get, for a point-like detector [34],

$$\begin{aligned} p^q(\mathbf{r},t) &\propto \int W(\{\alpha_{\mathbf{k}}\})\left[\mathcal{I}(\mathbf{r},t;\{\alpha_{\mathbf{k}}\})-I_0\right]d^{2N}\alpha_{\mathbf{k}} \\ &= \langle\mathcal{I}-I_0\rangle\ , \end{aligned} \tag{24}$$

where

$$\mathcal{I}(\mathbf{r},t;\{\alpha_{\mathbf{k}}\}) = c\epsilon_0 E^+(\mathbf{r},t;\{\alpha_{\mathbf{k}}\})E^-(\mathbf{r},t;\{\alpha_{\mathbf{k}}\}) \tag{25}$$

is the intensity for a realization of the field (14) at the position and time $(\mathbf{r},t)$, and $W(\{\alpha_{\mathbf{k}}\})$ is the Wigner function of the initial state. N is the number of modes (we shall later pass to the limit $N\rightarrow\infty$) and I_0 is the mean intensity of the ZPF. We use the calligraphic $\mathcal{I}$ for the random variable intensity in order to distinguish it from nonrandom averages like I_0. Then eq.(24) may be interpreted as stating that the detector has a threshold so that it only detects the part of the field which is above the average ZPF. Two remarks are in order: a) The intensity $\mathcal{I}$ contains a (possibly complicated) dependence on the initial amplitudes of all radiation modes, but I_0 is a constant (compare with (23)). We may interpret this by saying that the detector removes the ZPF. The quantum rule (24) is just to subtract the mean, a formal procedure which cannot be physical because it gives rise to "negative probabilities", as we shall discuss below. b) Strictly the integral in (24) should involve all radiation modes, but some cut-off frequency is required in order to avoid divergences. Furthermore, most of the radiation modes are usually not "activated" (in common quantum language we say that they contain no photons) and therefore they may be ignored, that is the contribution of these modes to the average $\langle\mathcal{I}\rangle$ equals the contribution to I_0. Ignoring them does not change the difference $\langle\mathcal{I}-I_0\rangle$. We shall call those which cannot be ignored the *relevant modes*.

In the same way we may obtain the coincidence detection probability for two detectors, placed at $(\mathbf{r}_1,t_1)$ and $(\mathbf{r}_2,t_2)$. We get from (14)

$$\begin{aligned} p_{12}^q(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2) &\propto \int W(\{\alpha_{\mathbf{k}}\})\left[\mathcal{I}\left(\mathbf{r}_1,t_1;\{\alpha_{\mathbf{k}}\}\right)-I_{10}\right] \\ &\quad\times\left[\mathcal{I}\left(\mathbf{r}_2,t_2;\{\alpha_{\mathbf{k}}\}\right)-I_{20}\right]d^{2N}\alpha_{\mathbf{k}} \\ &= \langle(\mathcal{I}_1-I_{10})(\mathcal{I}_2-I_{20})\rangle\ . \end{aligned} \tag{26}$$

In writing $I_{10}\neq I_{20}$ we are emphasizing that the thresholds of the two detectors may be different.

The relevant question for us is whether expressions (24) and (26) are suitable for a local realist interpretation. The answer is negative because $\mathcal{I}-I_0$ is not always positive and therefore cannot be interpreted as a probability. The problem is not the huge value of the zeropoint energy (the ZPF intensity is about 10^5w/cm^2 in the visible range), because the threshold intensity I_0 cancels precisely that intensity. The problem lies in the fluctuation of the intensity. For the weak light signals of typical quantum-optical experiments the fluctuations of $\mathcal{I}$ may be such that $\mathcal{I} < I_0$. Let us study whether this problem is real or is just a consequence of some approximations used in the standard quantum theory of detection.

We shall begin by showing that the removal of some idealizations involved in eqs. (24) or (26) alleviates the problem. Firstly these equations were derived including only modes corresponding to a beam of nearly parallel wave vectors. If this is not the case we should write eq.(26) using the Poynting vector rather than the intensity. The direction of the Poynting vector of the signal is well defined whilst that of the noise (the ZPF) is random with zero mean. This makes the discrimination easier. More important is the fact that the detection probability does not depend on the instantaneous intensity at a point. The quantum detection theory involves a hamiltonian which takes into account the interaction between the detector and the radiation field, whence the photon counting rate is calculated using first order perturbation theory. For the sake of simplicity let us consider a model detector consisting of a single atom, initially in the ground state. The interaction hamiltonian may be written

$$H_{\mathrm{int}} = \sum_j \sum_k \eta_{jk} \left(\widehat{\sigma}_{oj} \widehat{b}_k + \widehat{\sigma}_{jo} \widehat{b}_k^{\dagger} \right), \tag{27}$$

where $\widehat{\sigma}_{0j}$ ($\widehat{\sigma}_{j0}$) is the raising (lowering) operator for the atom to pass from the ground state 0 to the state j (from j to 0) and η_{jk} are some constants related to the detection efficiency. In the model a detection event, that is a *count*, happens when the atom becomes excited. The probability of a count is calculated using first order time-dependent perturbation theory and involves a time integral, whence one derives a probability of detection per unit time. For details see any book on quantum optics, e.g.[35].

The point is that the quantum theory of detection, derived from time-dependent perturbation theory, *necessarily* involves an integral over a finite (non-zero) time interval, T. The standard practice is to divide the counting probability by T in order to calculate the counting probability per unit time. However, in order to simplify the calculation, the limit $T \rightarrow \infty$ is used. Consequently the interval T does not appear in the quantum formula, but we stress that the standard procedure is accurate only if T is large. Also actual detectors are macroscopic bodies containing many atoms, and the probability of a count should involve a summation over all atoms, or equivalently a spatial integration. As a result, accurate quantum detection probabilities per unit time will not be of the form of eqs.(24) or (26), but should rather be given by

$$
\begin{aligned}
p_i^q &= \int W(\{\alpha_{\mathbf{k}}\})Q_i(\{\alpha_{\mathbf{k}}\},t)d^{2N}\alpha_{\mathbf{k}} \ , \ (i=1,2) \ , \\
p_{12}^q &= \int_t^{t+\Delta t} dt' \int W(\{\alpha_{\mathbf{k}}\})Q_1(\{\alpha_{\mathbf{k}}\},t)Q_2(\{\alpha_{\mathbf{k}}\},t')d^{2N}\alpha_{\mathbf{k}} \ , \\
Q_i &= \frac{\eta_i}{h\nu_i T}\int_{t-T}^{t} dt' \int d^2 r_i \left[\mathcal{I}_i(\{\alpha_{\mathbf{k}}\},\mathbf{r}_i,t') - I_{i0}\right] \ . \qquad (28)
\end{aligned}
$$

We stress that there are two different time intervals here, which should not be confused. Δt is a coincidence window defined by some electronic device with the purpose of recording counts of the second detector only during the time interval Δt after a count is produced in the first detector. On the other hand T is a time interval which appears when using time-dependent perturbation theory (in standard derivations the limit $T \rightarrow \infty$ is taken, as said above). In addition to the integration over T, we have included an integration over the surface aperture of the detector; the integration over the depth of the active detection zone being absorbed in the constant η_i. We have divided by the typical energy of one "photon" so that p_i^q and p_{12}^q may be identified with the measurable counting rates. Then η_i is the quantum efficiency of the i'th detector.

Q_i is a probability per unit time, and is therefore positive. It is defined by an integral over an interval previous to t in order to guarantee causal behaviour: the probability of a count within a small time interval around t should depend on the radiation entering the detector before (not after!) the time t.

The difficulties for a local realist theory of detection are alleviated by the time and space integrations in the definition of Q_i in (28). Indeed, the fluctuations of the intensity are strongly reduced by averaging over space-time regions, as the Heisenberg (uncertainty) relations show. But we can guarantee the positivity of Q_i only in the limit of infinitely wide detection time intervals. This is nonphysical but it corresponds precisely to the standard quantum procedure of taking $T \rightarrow \infty$.

There is still another correction required by a more accurate quantum theory of detection, namely going to higher orders of perturbation theory. This will certainly destroy the linearity of eqs.(28). Indeed these latter equations predict a counting probability which is linear in the intensity above the ZPF precisely because they use first order, that is linear, perturbation theory. It is frequently claimed that the linearity of the response is a straightforward consequence of the quantum theory of detection and that deviations from linearity in actual experiments are defects of the present technology of detectors. We see that, on the contrary, nonlinearity is a necessary feature of an accurate quantum theory of detection in which higher order perturbations are taken into account.

If we want to go beyond first-order perturbation theory, we need a more realistic model of a detector. Indeed we should consider at least a three-state – rather than a two-state – detection system if a count is to give rise to a *macroscopic* electric current. For instance, in a detector working via the photoelectric effect, the absorption of a photon liberates an electron which is then accelerated by an external potential. The collision of the accelerated electron with an appropriate device produces an avalanche giving rise to the macroscopic electric current. We have here three states: 1) the electron bound to the atom or to some part of a solid body, 2) the free electron, almost at rest, produced by photoelectric effect, and 3) the electron after being accelerated and after having lost some energy by collisions. A schematic picture of the potential seen by the electron is shown in Fig.2, where we represent the three states by horizontal lines. The point is that, in view of the above arguments, any accurate quantum theory of detection will predict a) A nonlinear response of the detector to the incoming radiation, and b) some nonthermal dark rate due to direct transitions from state 1 to state 3 by tunnelling.

The question is whether an accurate quantum theory of detection, when formulated in the Wigner representation, can be interpreted in a local realistic manner. We conjecture that this is the case. We shall not investigate it further here, though we have studied elsewhere [36] a three-state quantum detection model. Instead we shall simply give the essential properties of local realist detection models compatible with the assumption that the ZPF is real and has the same nature as the signals. A particular model along these lines has been proposed elsewhere [37] , based on an earlier one [29]. This will allow us to derive some general constraints which are not predicted by the standard linear quantum theory of detection, but which we conjecture would be predicted by a more accurate quantum treatment.

3.2 Local realist detection models

We now set out the basic points of any local realist model of detector for an incoming light beam having frequencies in the interval $(\omega_{\min}, \omega_{\max})$. If τ is the coherence time of the beam this means

$$\delta\omega \equiv \omega_{\max} - \omega_{\min} \approx 2\pi/\tau \ . \tag{29}$$

1. We shall assume that the detection probability depends on a quantity $\overline{\mathcal{I}}$ defined as a functional of the radiation entering the detector during some time interval $(t-T, t)$, that is

$$\overline{\mathcal{I}} = \overline{\mathcal{I}}\left[\mathbf{E}(\mathbf{r}, t'), \mathbf{B}(\mathbf{r}, t')\right] \quad , \quad \mathbf{r} \in \mathcal{V} \quad , \quad t' \in (t-T, t) \ , \tag{30}$$

where $\mathcal{V}$ is the relevant volume of the detector. The restriction of the domains of $\mathbf{r}$ and t give the locality condition. The quantity $\overline{\mathcal{I}}$ will be called the *effective intensity* and should be as close as possible to the actual intensity $\mathcal{I}$ of the radiation within the relevant modes. That is, the functional (30) should remove the radiation in most modes not carrying any signal, but retain the radiation coming in the relevant modes. Finding an appropriate functional is a difficult task (see, e.g. a particular functional in [37].)

2. We assume that the detection probability per unit time is given by eq.(28) but with the quantity Q replaced by the expression (from now on we consider, for simplicity, a single detector and consequently remove the subindex i)

$$\begin{aligned} Q(\overline{I}) &= T^{-1}(1 - \mathrm{e}^{-\zeta T(\overline{\mathcal{I}} - \overline{I}_0)})\Theta(\overline{\mathcal{I}} - I_m) \\ &= \zeta(\overline{\mathcal{I}} - \overline{I}_0)\Theta(\overline{\mathcal{I}} - I_m) + O(\zeta^2) \ , \end{aligned} \tag{31}$$

where $\overline{I}_0$ is the average of $\overline{\mathcal{I}}$ for the ZPF alone (when there is no signal present). I_m is some threshold intensity, related to the voltage bias of the detector and fulfilling the condition $I_m > \overline{I}_0$, and $\Theta(x)$ is the Heaviside function, $\Theta(x) = 1$ if $x > 0$, 0 otherwise. Our choice guarantees that the probability per unit time Q satisfies the positivity condition $Q \geq 0$. We assume an exponential function, rather than a linear dependence on $\overline{\mathcal{I}} - \overline{I}_0$, in order to prevent Q becoming greater than one. The definition of $\overline{\mathcal{I}}$ as a functional over a finite (nonzero) time interval implies that two counts cannot be produced within a time interval smaller than T. This implies that T should be smaller than the dead time of the detector. The parameter ζ is proportional to the detector efficiency but has dimensions of area divided by energy.

This completes the definition of our class of models. Now we shall rewrite the detection probability (28) in an equivalent form by introducing an additional integration with a Dirac's delta, $\delta(\overline{\mathcal{I}} - J)$. That is (the superindex m is for "model")

$$p^m = \int W(\{\alpha_{\mathbf{k}}\}) Q(\overline{\mathcal{I}}) \delta[\overline{\mathcal{I}} - J(\{\alpha_{\mathbf{k}}\}, \phi, t')] d\overline{\mathcal{I}} d^{2N}\alpha_{\mathbf{k}} \ , \tag{32}$$

where we include in $J(\{\alpha_{\mathbf{k}}\}, \phi, t)$ all the dependence of the effective intensity $\overline{\mathcal{I}}$ on the initial field amplitudes $\{\alpha_{\mathbf{k}}\}$ and the controllable parameters ϕ of the experiment. In principle it is possible, although cumbersome, to perform the integration over the amplitudes $\{\alpha_{\mathbf{k}}\}$ in (32), and rewrite it in the compact form

$$p^m = \int \rho(\overline{\mathcal{I}}, t) Q(\overline{\mathcal{I}}) d\overline{\mathcal{I}} \ , \tag{33}$$

$$\rho(\overline{\mathcal{I}}, t) = \int W(\{\alpha_{\mathbf{k}}\}) \delta[\overline{\mathcal{I}} - J(\{\alpha_{\mathbf{k}}\}, \phi, t)] d^{2N}\alpha_{\mathbf{k}} \ . \tag{34}$$

Similarly we obtain, for the joint detection probability,

$$p_{12}^m = \int_t^{t+\Delta t} dt' \int \rho_{12}(\overline{\mathcal{I}}_1, \overline{\mathcal{I}}_2, t, t') Q_1(\overline{\mathcal{I}}_1) Q_2(\overline{\mathcal{I}}_2) d\overline{\mathcal{I}}_1 d\overline{\mathcal{I}}_2 \ , \tag{35}$$

$$\begin{array}{c} \rho_{12}(\overline{\mathcal{I}}_1, \overline{\mathcal{I}}_2, t, t') = \int W(\{\alpha_{\mathbf{k}}\}, \{\alpha_{\mathbf{k}}^*\}) \delta[\overline{\mathcal{I}}_1 - J_1(\{\alpha_{\mathbf{k}}\}, \phi_1, t)] \\ \times \delta[\overline{\mathcal{I}}_2 - J_2(\{\alpha_{\mathbf{k}}\}, \phi_2, t'')] d^{2N}\alpha_{\mathbf{k}} \end{array} \ . \tag{36}$$

These two expressions are very convenient for the comparison between our model and the standard (linear) quantum treatment. The dependence of J on $\{\alpha_{\mathbf{k}}\}$ should be derived from the quantum Wigner formalism for every specific experiment. For instance this was done in Refs. [34, 42, 43, 44] for most of the spontaneous parametric down-conversion (SPDC) experiments performed to date.

Now we shall study whether the predictions of the local realist models above sketched are compatible with the results of performed experiments, and the extent to which they agree with the quantum predictions. For that purpose the following steps will be taken: *i)* To obtain the probability distribution, $\rho_0(\overline{\mathcal{I}})$, of the effective intensity of the ZPF in absence of any further electromagnetic radiation. *ii)* To obtain the probability distribution, $\rho(\overline{\mathcal{I}})$, for the radiation that arrives at the detector when it is illuminated with a light beam.

We may assume that the probability distribution of the effective intensity when only the ZPF is present is gaussian, that is

$$\rho_0(\overline{\mathcal{I}}) = \frac{1}{\sigma_0\sqrt{2\pi}} \mathrm{e}^{-(\overline{\mathcal{I}}-\overline{I}_0)^2/2\sigma_0^2} \ . \tag{37}$$

The calculation of the probability distribution of effective intensity when there is a signal present depends on the nature of the signal. If it is gaussian and stationary, which happens for instance in SPDC (see next section), we get

$$\rho(\overline{I}) \cong \frac{1}{\sigma_0\sqrt{2\pi}}\, e^{-(\overline{\mathcal{I}}-\overline{I}_s-\overline{I}_0)^2/2\sigma_0^2} \ , \tag{38}$$

where we have defined the signal effective mean intensity by

$$\overline{I}_s \equiv \langle \overline{\mathcal{I}} \rangle - \overline{I}_0, \tag{39}$$

and have assumed that $\overline{I}_s \ll \overline{I}_0$ in the numerator of the exponent, so that the variance of $\overline{\mathcal{I}}$ is effectively σ_0^2.

Hence we obtain the detection probability within a window, using eqs.(38), (33) and the approximate expression in (31). We get

$$p^m = \frac{1}{2}\, \zeta \overline{I}_s \,\mathrm{erfc}(-z) + \frac{\zeta\sigma_0}{\sqrt{2\pi}}\, e^{-z^2} + O(\zeta^2) \ , \tag{40}$$

where

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp\left(-t^2\right) dt \ ,$$

and

$$z = (\overline{I}_s + \overline{I}_0 - I_m)/(\sigma_0\sqrt{2}) \ .$$

This expression clearly shows the nonlinear dependence of the detection probability on the incoming intensity. However, if

$$\overline{I}_s \gg \sigma_0 \ , \tag{41}$$

then choosing the controllable parameter I_m, related to the voltage bias, so that

$$\overline{I}_0 + \overline{I}_s - I_m \gg \sigma_0 \ , \tag{42}$$

(but $I_m > \overline{I}_0$ in order to preserve the positivity of Q_i in eq. (31)) we obtain

$$P^m \cong \zeta \overline{I}_s \ . \tag{43}$$

This is very similar to the standard (linear) quantum result, the only difference being the presence of the effective intensity of the signal, $\bar{I}_s$, instead of the instantaneous intensity I_s, in the quantum formula. If they are proportional, full agreement may be obtained by appropriate choice of ζ. We shall therefore refer, in section 5 of the present article, to eq.(24) as the *linear quantum detection theory*, and it may legitimately be considered as a linear approximation to a local realist theory of the type we have just described. But we stress that the approximation is not valid for very weak signals ($\overline{I}_s = O(\sigma_0)$) or for high detector efficiency ($\zeta\overline{I}_s = O(1)$), the latter because the approximation in (31) is not valid.

We stress that we have been speaking in this section only about "photon counters", which are the detectors typically used in quantum optics. The detectors used in astronomy, which are also the type needed for the SPUC experiment to be described in the next section, use a long resolution time. Such detectors simply measure the total energy, above the ZPF, which is incident upon them, so the standard normal-ordering theory of detection, based on $T \to \infty$, may be used.

3.3 Constraints put on detection by local realism

The predictions of our model agree with those of standard quantum theory for experiments in which the intensity of the signal is not too weak, so that (41) holds true, and the detector's efficiency is low enough, so that the linear approximation in (31) may be used. In particular we conjecture that local realist models exist which are compatible with the violation of all Bell inequalities tested experimentally to date, those inequalities having been derived from local realism *plus auxiliary assumptions.* At high efficiency, the model departs from conventional quantum theory, in that it predicts a nonlinear response of the detectors (see eqs.(32) and (31)), or a high dark rate, or both (see eq.(40)). This is the feature that would prevent the violation of a genuine (inhomogeneous) Bell inequality, that is one involving no auxiliary assumptions in addition to local realism. Our model would be disproved if an optical experiment violated a genuine Bell inequality, that is what is usually named a "loophole-free test". But apparently the possibility of such a test still lies far in the future [17]. In any case, such a test would involve measurements of both singles and coincidence counts [38], and also should avoid any background subtraction. The latter condition derives from the fact that our model predicts the existence of some fundamental dark rate in photon detectors, and there is no reason why a Bell inequality should be satisfied if that rate is subtracted.

For particular models more simple tests may be possible. In these models it would be possible to calculate the parameters σ_0 and ζ defined in eqs.(31) and (37) in terms of parameters which are measurable plus the parameter T, which has the dead time of the detector as upper bound. This gives inequalities which may be empirically tested. In this form any particular model would have predictive power going beyond standard quantum optics. For instance, nobody will claim that quantum mechanics is violated by an experiment if it is discovered that there is a dark rate or that the response of the detector to the signal is nonlinear, both features in disagreement with the predictions of the linear quantum theory of detection. These facts will be attributed to imperfect functioning of the detectors, imperfections which are considered technological problems irrelevant for the test of quantum theory. In contrast, our models indicate rather stringent and fundamental constraints on the functioning of detectors precisely because the ZPF is taken as real. And the reality of the ZPF is an unavoidable consequence of assuming that the Wigner function is a probability distribution, which is the central idea of our approach.

For illustrative purposes we give the inequality which has been derived in our model of [37]. It predicts a lower bound for the singles counting rate

$$\text{Rate} \gg \frac{\eta\lambda f^2}{2R_l^2 L\sqrt{\tau T}}, \tag{44}$$

where η is the quantum efficiency of the detector, λ the typical wavelenght of the signal, R_l and f are the radius and focal length of the collecting lens, L the depth of the active zone of the detector and τ the coherence time of the signal beam. That particular model seems to be incompatible with some recent experimental results [39], but we expect to find more refined models which are compatible with these experiments. In any case we conjecture that the existence of a minimum detectable intensity is model-independent; it is an unavoidable consequence of taking the ZPF as real. But in order to avoid misunderstanding we stress again that the intensity capable of being detected may be as small as we want in the case of long time windows, that is large T, or if we have highly monochromatic signals, i.e. large τ, as is shown by (44) . This fits with the fact that the standard, linear quantum theory of detection is derived in the limit $T \rightarrow \infty$, as discussed above.

4 Spontaneous parametric up and down conversion

We have drawn attention to the two problems which a local realist interpretation of the optical Wigner function must confront. In Section 2 we considered the first of these, which is that there are quantum states, for example the number states, which give nonpositive Wigner functions. In section 3 we considered the other one, which is that a simple subtraction of the ZPF, by normal ordering, produces negative detection probability for some signals. We have shown that both problems have solutions. Indeed, in each case we have found that the solution depends on recognizing the multimode nature of the signal. In the present section we shall confine attention to signals produced by the process of spontaneous parametric down conversion (SPDC) in nonlinear crystals. This has been the area most intensively researched, during the last two decades, in connection with photon-entanglement phenomena, which we discuss in the next section. It is also the area most adequately treated in the Wigner function formalism, because it is possible to treat the production of the signal entirely classically, leaving us with only the detection problem. Our treatment of SPDC leads us to the prediction of a new phenomenon, spontaneous parametric up conversion (SPUC), which we analyze in the latter part of this section.

4.1 Down conversion

Parametric Down Conversion (PDC) is the optical analogue for a well established classical phenomenon of wave propagation in a nonlinear dispersive medium. It was discovered in the 1960s, when high-intensity coherent sources (lasers) became available. When two lasers of frequencies ω_1 and ω_3 ($\omega_1 > \omega_3$) are incident on a nonlinear crystal (NLC) whose space group has no centre of symmetry, then, for a certain combination of incidence angles, a signal of frequency $\omega_2 = \omega_1 - \omega_3$ is emitted (see Fig.3).

For example ω_1 could be a normally incident laser at 351nm and ω_3 could be another laser at 845nm, giving the wavelength of ω_2 at 600nm. The angles θ_3 and θ_2 are given by certain phase matching relations between the wave modes within the crystal, and depend on its refractive indices. In a BBO crystal, cut so that the optic axis makes an angle of 37 degrees with ω_1, they are θ_3= 11.4 degrees and θ_2= 8.1 degrees. Note that the incident ω_1 wave has extraordinary, and the ω_3 wave ordinary polarization. The signal at ω_2 is then ordinarily polarized, and the process is known as Type-I PDC.

SPDC is the name given to the phenomenon which occurs when we remove the laser ω_3; a weak signal, which is nevertheless visible to the unaided eye, remains in the outgoing ω_2 channel. This is because, in the vacuum, there is a ZPF intensity in all modes, corresponding to half a "photon". The SPDC phenomenon actually manifests itself as a rainbow (see Fig.4); since all frequencies and directions of modes are present in the ZPF, all the corresponding down converted signals also appear at the angles satisfying the appropriate phase matching relations. We shall discuss below the way in which the exit angles of the rainbow are determined.

The above phenomenon is nowadays often called simply PDC, because it has become popular to view the process as one in which laser "photons" ω_1 down convert into ω_2 and ω_3. In this description the phase matching relations linking the wave vectors of the three coupled modes in SPDC are considered to express conservation of four-momentum between the three participating "photons". It is an unfortunate, and highly misleading, consequence of this description that certain correlations in the intensities of the outgoing SPDC signals are now widely interpreted as showing rather bizarre connections between the corresponding "photons". The related phenomenon of SPUC, which we describe later in this section, will show that the wave description of SPDC, which we gave above, is the more correct one.

The electric-field operator in a nonlinear crystal satisfies the Maxwell equation[40]

$$\nabla \times \nabla \times \hat{\mathbf{E}} + \frac{1}{c^2}\,\frac{\partial^2(\epsilon \cdot \hat{\mathbf{E}})}{\partial t^2} = -\mu_0 \frac{\partial^2 \hat{\mathbf{P}}^{\mathrm{NL}}}{\partial t^2}\ , \tag{45}$$

where $\hat{\mathbf{P}}$ is the polarization vector, whose linear part is contained within the permittivity tensor ϵ, and whose nonlinear part is given by

$$\hat{P}_i^{\mathrm{NL}} = 2 d_{ijk} \hat{E}_j \hat{E}_k\ , \tag{46}$$

where d_{ijk} is essentially the Pockels tensor[41]. We have linearized this equation[40] by separating the part of $\hat{\mathbf{E}}$ represented by the pumping laser

ω_1, that is by making the replacement

$$\hat{\mathbf{E}}(\mathbf{r},t) \to 2A_1\mathbf{u_1}\cos[\omega_1(t - n_1 z/c)] + \hat{\mathbf{E}}(\mathbf{r},t) \ , \tag{47}$$

where n_1 is the refractive index of the extraordinary wave ω_1 propagating in the z, that is normal, direction indicated in Fig.3, and $\mathbf{u}_1$ is a unit vector in the direction of that mode's electric vector. Note that, since this mode is extraordinary, $\mathbf{u}_1$ has a nonzero z-component, and that n_1 is an appropriate square root of the tensor ϵ. Thus we have essentially abstracted the field component corresponding to the pump from (15). We are also neglecting the fluctuations in the pump's amplitude, so we replace the operator $\hat{a}_1$ in (15) by the "variable" α_1 of (14), which is taken as constant in our linearized procedure. Then A_1 in the above equation is related to α_1 by

$$A_1 = \frac{\hbar c|\mathbf{k}_1|}{2\epsilon_0 L^3}\alpha_1 \ . \tag{48}$$

Now because the equation determining $\hat{\mathbf{E}}$, in our approximation, is linear, and because the unperturbed state of the field is the gaussian process given by the Wigner function of the vacuum, we have been able to show (see the above reference) that *all of the other field amplitudes may now be replaced by their stochastic equivalents*, that is we may simply use the field defined by (14), and it satisfies the stochastic differential equation

$$\begin{aligned}
[\nabla \times \nabla \times \mathbf{E}]_i + \frac{\epsilon_{ij}}{c^2}\ \frac{\partial^2 E_j}{\partial t^2} = \\
8\mu_0 A_1^2 d_{ijk} u_{1j} u_{1k} \sin[2\omega_1(t - n_1 z/c)] \\
-4\mu_0 A_1 d_{ijk} u_{1j} \frac{\partial^2 E_k \cos[\omega_1(t - n_1 z/c)]}{\partial t^2} \ .
\end{aligned} \tag{49}$$

We have therefore shown that solving this equation, with the initial values of all the field amplitudes given by the distribution (6), will give exactly the same result as solving the operator equation (45). Note that we have discarded the terms in $\mathbf{P}^{(\mathrm{NL})}$ which are quadratic in the vacuum part of the field; this is the essence of our linearization.

We have followed two procedures for solving eq.(49). The first is close to the quantum optical procedure, and consists in using the hamiltonian to deduce the time derivative of $\alpha_{\mathbf{k},\lambda}$[34, 42, 43, 44]. The second is to work with the stochastic equation directly, either by a perturbation procedure[45, 46] or by a mode-coupling one, based originally on that described in Ref.[41] Section 19.4 and developed by us in Ref.[40], and we shall use the latter here. We look for a solution in which $\mathbf{E}(\mathbf{r},t)$ is given by eq.(14), but the amplitudes $\alpha_{\mathbf{k},\lambda}$ are slowly varying functions of z. Then the two ordinary (that is $\lambda = 1$) modes ω_2 and ω_3 are coupled

$$\begin{aligned} \frac{d\alpha_2}{dz} &= iG_2 e^{-i\Delta z}\alpha_3^* \ , \\ \frac{d\alpha_3}{dz} &= iG_3 e^{-i\Delta z}\alpha_2^* \ , \end{aligned} \tag{50}$$

where

$$\begin{aligned} \Delta = k_{1z} - k_{2z} - k_{3z} \ , \ G_2 = d_{ijk}u_{2i}u_{3j}u_{3k}A_1\frac{\omega_2^2}{k_{2z}} \ , \\ G_3 = d_{ijk}u_{3i}u_{1j}u_{2k}A_1\frac{\omega_3^2}{k_{3z}} \ , \end{aligned} \tag{51}$$

and our model imposes perfect phase matching in the x- and y-directions and in frequency, that is

$$k_{1x} - k_{2x} - k_{3x} = 0 \ , \ k_{1y} - k_{2y} - k_{3y} = 0 \ , \ \omega_1 - \omega_2 - \omega_3 = 0 \ . \tag{52}$$

The solution to these coupled equations is

$$\begin{aligned} \alpha_2(z) = \left[\cos(\tfrac{1}{2}Kz) + \tfrac{i\Delta}{K}\sin(\tfrac{1}{2}Kz)\right] e^{-i\Delta z/2}\alpha_2(0) \\ + \tfrac{2iG_2}{K} \ \sin(\tfrac{1}{2}Kz)e^{-i\Delta z/2}\alpha_3^*(0) \ , \end{aligned} \tag{53}$$

where

$$K^2 = \Delta^2 - 4G_2G_3 \ , \tag{54}$$

with a similar expression for $\alpha_3(z)$.

The intensities in the outgoing SPDC beams are obtained by defining filtered components of the outgoing electric field $\mathbf{E}$, designated $\mathbf{E}_2$ and $\mathbf{E}_3$. They represent narrow subsets of wave vectors close to $\mathbf{k}_2$ and $\mathbf{k}_3$, which, as discussed in the previous section, are the parts of the emitted field sampled at the "photon" detectors. We define also the positive and negative frequency parts of these fields $\mathbf{E}_i^{(+)}$ and $\mathbf{E}_i^{(-)}$ in the same way as in eq.(14). For example

$$\mathbf{E}_2^{(+)}(\mathbf{r},t) = \mathbf{u}_2\sqrt{\frac{\hbar|\mathbf{k_2}|}{2\epsilon_0 L^3}}\sum_{\mathbf{k}}^{(2)}\alpha_{\mathbf{k},1}(l)e^{i\mathbf{k}.\mathbf{r}-i|\mathbf{k}|ct/n} \ , \tag{55}$$

where the index (2) on the summation denotes the above process of filtration for beam (2), and it is limited to the ordinary modes ($\lambda = 1$). Then the outgoing intensities in the two beams are obtained by summing the squares of the above mode amplitudes, that is

$$I_{2,3}(l) = \frac{\hbar\omega_{2,3}}{2L^3} \sum^{(2+3)} \langle \alpha_{2,3}(l)\alpha^*_{2,3}(l)\rangle \ , \tag{56}$$

where l is the length of the crystal and $\langle\rangle$ denotes an averaging process weighted by the vacuum distribution (6). The (2+3) over the summation indicates that we are summing over all *pairs* of modes, in the (2) and (3) subsets, which satisfy the phase-matching conditions (52). We find that

$$I_2(l) - I_2(0) = \frac{\hbar\omega_2 l^2}{4L^3}\, G_2(G_2 + G_3) \sum^{(2+3)} \mathrm{sinc}^2(\tfrac{1}{2}\Delta l) \ , \tag{57}$$

with a similar expression for the other mode. We have here introduced the sinc-function, defined by

$$\mathrm{sinc}(x) = \frac{\sin x}{x} \ , \tag{58}$$

and we have approximated by putting K equal to Δ, which means these expressions are correct to order G^2; to this approximation the expressions obtained with the alternative perturbation approach[45, 46] are the same.

We see from the above result that all ordinary modes of the ZPF are amplified above their vacuum values by the nonlinear interaction. There is nothing miraculous about this process; the increase in their energy is at the cost of a minuscule depletion of the laser amplitude A_1. The greatest amplification is in those modes for which the sinc-function has a maximum, that is for modes giving Δ close to zero, which means perfect phase matching in the z-direction (or conservation of "photon z -momentum"). By reference to the previous section, we see that a detector with a long resolving window, like the human eye, simply subtracts the vacuum intensity level in all modes. So the position of the visible rainbow is given by those values of $\mathbf{k}_2$ giving perfect phase matching in time and in all three spatial directions, that is

$$\begin{aligned} \mathbf{k}_1 &= \mathbf{k}_2 + \mathbf{k}_3 \ , \\ |\mathbf{k}_1|/n_1 &= |\mathbf{k}_2|/n_2 + |\mathbf{k}_3|/n_3 \ , \end{aligned} \tag{59}$$

where n_1 is the refractive index for an extraordinary wave at ω_3, and n_2, n_3 are the indices for ordinary waves at ω_2, ω_3. This is all we needed in order to construct Fig.4.

In addition, the modes $\mathbf{k}_2$ and $\mathbf{k}_3$ are strongly *correlated*, that is large values of $I_2(l)$ are associated, through the coupling, with large $I_3(l)$. This explains qualitatively the synchronization observed in photon counts[47, 48], and also the body of SPDC data which is widely interpreted as showing *photon entanglement*. To discuss these data requires that we apply the detection theory developed in the preceding section to short windows, of the order 10ns, which we shall do in the following section. Some results which follow directly from our analysis[40], and which we shall use in the sequel, are the electric field autocorrelation in the outgoing beam, and the cross correlation between two "entangled" beams. The field autocorrelation for the beam (2) is obtained by taking the dyadic product $\mathbf{E}_2(\mathbf{r}, t)\mathbf{E}_2(\mathbf{r}', t')$ averaged over the random variables $\alpha_{\mathbf{k},\lambda}(0)$, using the vacuum Wigner distribution. This is a generalization of the procedure we just used to calculate the beam intensity. We obtain

$$\langle \mathbf{E}_2^{(+)}(\mathbf{r}, t)\mathbf{E}_2^{(+)}(\mathbf{r}', t')\rangle = \langle \mathbf{E}_2^{(-)}(\mathbf{r}, t)\mathbf{E}_2^{(-)}(\mathbf{r}', t')\rangle = 0 \ ,$$
$$\langle \mathbf{E}_2^{(+)}(\mathbf{r}, t)\mathbf{E}_2^{(-)}(\mathbf{r}', t')\rangle - \langle \mathbf{E}_2^{(+)}(\mathbf{r}, t)\mathbf{E}_2^{(-)}(\mathbf{r}', t')\rangle_0 =$$
$$\frac{\hbar\omega_2 l^2}{4L^3}\ G_2(G_2+G_3)\ \mathbf{u}_2\mathbf{u}_2 \sum^{(2+3)} \mathrm{sinc}^2(\tfrac{1}{2}\Delta l)e^{i\mathbf{k}_2\cdot(\mathbf{r}-\mathbf{r}')-i|\mathbf{k}_2|c(t-t')} \ . \qquad (60)$$

The latter expression may be written as

$$\mathbf{u}_2\mathbf{u}_2\ [I_2(l) - I_2(0)]\ \mu_2(\mathbf{r}-\mathbf{r}', t-t') \ ,$$

where $\mu_2(\mathbf{0}, 0) = 1$, and the autocorrelation for $\mathbf{k}_3$ is given by a similar expression. It will be observed that $\mu_{2,3}(\mathbf{r}, t) \to 0$ as either $|\mathbf{r}| \to \infty$ or $|t| \to \infty$. These properties enable us to define a coherence time and length for the SPDC beams. The cross correlation between the two beams is

$$\langle \mathbf{E}_2^{(+)}(\mathbf{r}, t)\mathbf{E}_3^{(-)}(\mathbf{r}', t')\rangle = \langle \mathbf{E}_2^{(-)}(\mathbf{r}, t)\mathbf{E}_3^{(+)}(\mathbf{r}', t')\rangle = 0 \ ,$$
$$\langle \mathbf{E}_2^{(+)}(\mathbf{r}, t)\mathbf{E}_3^{(+)}(\mathbf{r}', t')\rangle = \langle \mathbf{E}_2^{(-)}(\mathbf{r}, t)\mathbf{E}_3^{(-)}(\mathbf{r}', t')\rangle =$$
$$\frac{\hbar\sqrt{\omega_2\omega_3}}{4L^3}\ (G_2+G_3)\ \mathbf{u}_2\mathbf{u}_3 \sum^{(2+3)} l\,\mathrm{sinc}(\Delta l)e^{i\mathbf{k}_2\cdot\mathbf{r}+i\mathbf{k}_3\cdot\mathbf{r}'-ic(|\mathbf{k}_2|t+|\mathbf{k}_3|t')-i\Delta l} \ . \qquad (61)$$

4.2 Up conversion

The phenomenon of spontaneous parametric up conversion (SPUC)[40, 49] may be understood by seeing what happens when we reverse the roles of the pump and the ZPF mode in Fig.3, that is the coherent input ω_1 is coupled to a ZPF mode ω_3 with $\omega_3 > \omega_1$. Then, by the same process as before, a signal of frequency $\omega_3 - \omega_1$ may be emitted, and, if the angles are such that the phase matching relations are satisfied, this signal will have an intensity approximately the same as was observed in SPDC, that is it will be visible to the unaided eye. What is remarkable is that, if $\omega_3 > 2\omega_1$ then $\omega_2 > \omega_1$, which means that the signal "photon" has a higher energy than a "photon" of the pumping laser, which is why we call the phenomenon *up* conversion.

It is a simple matter to calculate the position of the SPUC rainbow, along the same lines as we used to draw Fig.4. This has been discussed qualitatively elsewhere[49]. We find that, with the same BBO crystal as was used for Fig.4, the SPUC rainbow has the form depicted in Fig.5.

As expected, we find that the phase matching relations are satisfied, with a pump (ω_1) at 845nm which has ordinary polarization, for a wide range of visible frequencies (ω_2). These outgoing visible signals result from the mixing of ZPF modes in the ultraviolet with the infrared pumping mode. In contrast with SPDC, the rainbow is not symmetrical; the value of θ_2 depends on the azimuthal angle ϕ_2 which it makes with the crystal's optic axis. Indeed, when we come to consider the intensity we shall see that, for all visible frequencies, the rainbow is very much more intense in the region of $\phi_2 = 180$ degrees. The reason for the anisotropy is that the partner mode of the ordinarily polarized ω_2 is an extraordinarily polarized ω_3, whose refractive index depends on the angle its wave vector makes with the optic axis.

To calculate the intensity, we have to sum the individual mode intensities, given by a calculation[40] almost identical with what we described for SPDC in eq.(57). The resulting intensity is a function of the frequency ω_2 and of the azimuthal angle ϕ_2, and contains the intensity of the incident laser $|A_1|^2$ as a factor. Dividing by this factor gives the *cross section* for the SPUC process, and this may be compared with the cross section of a typical SPDC process. We have taken as examples the SPDC process from a pump at 442nm and a SPUC process with a pump at 845nm. In Fig.6 we plot the two cross sections, as function of the azimuthal angle for $\lambda_2 = 600$nm (left), and as a function of wavelength at azimuth 180 degrees (right).

The first plot shows a strong concentration of intensity for SPUC at azimuths near to 180 degrees; this contrasts with a fairly uniform azimuthal distribution in SPDC, and, together with the exiting angle depicted in Fig.5, it tells us both the position and intensity of the SPUC rainbow. The right plot shows that, at 180 degree azimuth, the intensity of the SPUC rainbow, with a normally incident laser at 845nm, is about half that of the SPDC rainbow with a normally incident 442nm laser. It is therefore not at all a difficult matter to observe the SPUC rainbow, and indeed to photograph it in glorious technicolor!

5 What is entanglement?

Schrödinger[50] introduced the word *entanglement* for what he described as "not *one* but rather *the* characteristic trait of quantum mechanics". Entangled states are states of two or more particles that cannot be written, in the hilbert-space formalism, as products of single-particle states. The paradigm of entanglement is the state described by EPR[1] which, in the case of polarization-entangled photon pairs is

$$| \Psi\rangle = \frac{1}{\sqrt{2}} \left(| R1\rangle \mid R2\rangle - \mid L1\rangle \mid L2\rangle \right), \tag{62}$$

where R (L) means a right (left) hand circular polarization. In the following we shall show that entanglement has a very simple explanation in the Wigner formalism of quantum optics: it is just a correlation which involves the ZPF, whilst "classical" correlations involve only the radiation which is above the level of the ZPF.

Entanglement gives rise to most of the allegedly nonclassical quantum phenomena, and has received considerable attention in the last few decades. In particular it is at the basis of a new branch of quantum theory called quantum information which is being actively investigated at present[51]. The most dramatic consequence of entanglement is that it predicts violation of local realism, supposedly demonstrated in tests of the Bell inequalities. In the next subsection we shall see that the action of a beam splitter provides a simple example of entanglement, and shall show how the paradoxical wave-particle behaviour of light may be explained easily by a purely wave theory.

5.1 The action of a beam splitter and the wave-particle duality

In classical optics a beam splitter usually has an incoming channel, say channel 1, and two outgoing channels, say 2 and 3 (see Fig.7). If a light beam arrives at channel 1, a fraction of the intensity goes to 2 and other fraction to 3. For simplicity we shall consider balanced beam splitters where half of the intensity goes to each outgoing channel.

In the hilbert-space formalism of quantum optics the action of the beam splitter is described by stating that, if one photon arrives in channel 1, the state in channels 2 and 3 is represented by the entangled state

$$\mid \Psi \rangle = \frac{1}{\sqrt{2}} \left(\mid 1_2 \rangle \mid 0_3 \rangle + \mid 0_2 \rangle \mid 1_3 \rangle \right), \tag{63}$$

where $\mid 1 \rangle$ ($\mid 0 \rangle$) means that 1 (0) photon is present in the channel labelled by the subindex. This represents a linear combination of two states, each with one photon in one channel and the vacuum in the other channel. If we calculate the probability of getting one photon in channel 2 we obtain, according to the standard quantum measurement theory

$$\langle \Psi \mid \hat{a}_2^\dagger \hat{a}_2 \mid \Psi \rangle = \tfrac{1}{2}, \tag{64}$$

where $\widehat{n}_2 = \hat{a}_2^\dagger \hat{a}_2$ is the observable "number of photons in channel 2". Similarly, the probability of getting one photon in channel 2 and another one in channel 3 is

$$\langle \Psi \mid \hat{a}_2^\dagger \hat{a}_3^\dagger \hat{a}_2 \hat{a}_3 \mid \Psi \rangle = 0. \tag{65}$$

This is interpreted as saying that the incoming photon goes undivided into one of the channels, which allegedly proves the corpuscular nature of the photon. However, if the beams 2 and 3 are recombined at another beam splitter it is possible to show interference, which clearly indicates wave properties of light. An experiment resting upon these quantum optical predictions was performed by Grangier et al.[23]. The experiment will be described and analyzed below in some detail. These predictions are considered an example of the counterintuitive and highly nonclassical phenomena associated to quantum entanglement. But we shall see in the following that a rather intuitive picture may be obtained from the analysis of the phenomena in the Wigner formalism.

As usual, the Wigner formalism of quantum optics treats light exactly as classical optics, derived from Maxwell equations, except that it includes a ZPF. In particular, if we are interested in the radiation going out from channels 2 and 3 we should include, in addition to the signal beam entering by channel 1, an incoming ZPF radiation entering by the fourth channel which we shall label 0. Thus, in the Heisenberg picture of the Wigner representation, we shall write

$$E_2^{(+)}(t) = \frac{1}{\sqrt{2}}\left(E_0^{(+)}(t) + iE_1^{(+)}(t)\right), \tag{66}$$

$$E_3^{(+)}(t) = \frac{1}{\sqrt{2}}\left(E_1^{(+)}(t) + iE_0^{(+)}(t)\right), \tag{67}$$

and similarly for $E_2^{(-)}(t)$ and $E_3^{(-)}(t)$. We remember that $E_1^{(+)}(t)$ is given, in terms of the amplitudes of the modes, by eq.(14) but including in the summation the relevant modes only (i.e. those where there is radiation above the ZPF). In $E_0^{(+)}(t)$ we include only the relevant modes, defined as those which could interfere with the ones of $E_1^{(+)}(t)$ in the outgoing channels. In particular the beam represented by $E_0^{(+)}(t)$ should have the same polarization as that of $E_1^{(+)}(t)$ and this is why we use a scalar representation in (66). We note that the need to take account of the fourth beam-splitter channel is recognized in the hilbert-space formalism, where that input is required in order preserve the commutation relations between the field operators [52].

From (66) it is easy to calculate the singles (R_j) and coincidence (R_{23}) counting rates at detectors in beams 2 and 3. These quantities are more physical than the probabilities (64) and (65). In the linear detection theory (see section 3) we have

$$R_3(t) = R_2(t) = \eta\left\langle I_2(t) - I_0\right\rangle\ ,$$
$$R_{23}(t,t';\tau) = \eta^2 \int_{t'}^{t'+\tau} dt'' \left\langle \left(I_2(t) - I_0\right)\left(I_3(t'') - I_0\right)\right\rangle\ , \tag{68}$$

where, for $i = 2, 3$,

$$I_i(t) = c\epsilon_0 E_i^{(+)}(t) E_i^{(-)}(t)\ , \tag{69}$$

τ is the coincidence detection window and η is a constant proportional to the efficiency. In the calculation of (68) we shall use the property that for four gaussian amplitudes, say A, B, C, D, the following equality holds

$$\langle ABCD\rangle = \langle AB\rangle \langle CD\rangle + \langle AC\rangle \langle BD\rangle + \langle AD\rangle \langle BC\rangle . \tag{70}$$

Thus, taking into account that $E_0(t)$ is uncorrelated with $E_1(t)$, we have

$$\begin{aligned} R_2 (t) &= \tfrac{1}{2}\eta(\langle I_1 (t)\rangle + c\epsilon_0\langle E_0^{(+)}(t)E_0^{(-)}(t)\rangle - 2I_0) \\ &= \tfrac{1}{2}\eta\langle I_1 (t) - I_0\rangle \equiv \tfrac{1}{2}\eta\mathcal{I}_1 (t) , \end{aligned} \tag{71}$$

the latter equality coming from the identification $c\epsilon_0 \left\langle E_0^{(+)}(t)E_0^{(-)}(t)\right\rangle = I_0$, and $\mathcal{I}_1$ is the expectation value of the intensity (above ZPF) of the signal beam entering by channel 1. The coincidence rate depends on the nature of the signal beam. For instance, if it is produced by SPDC, the calculation is straightforward, taking into account its gaussian character and the autocorrelation (60) of $E_1^{(+)}(t)$. We get

$$R_{23} (t, t'; \tau) = \frac{1}{4}\eta^2\mathcal{I}_1^2 \left[\tau + \int_0^\tau dt''|\mu (t'') |^2\right] , \tag{72}$$

where μ is the function defined in (60) for $\mathbf{r} = \mathbf{r}'$.

We see that, at $t' = t$, the coincidence probability is greater (almost twice for small τ) than the one expected if the incoming beam had a nonfluctuating intensity. (In actual experiments that result may be observed only if the beams have sufficient spatial coherence.) The fact that $R_{23} > \tau R_2R_3$ is usually called *photon bunching* and it is explained, in standard quantum theory, as a consequence of the boson character of the photons. In the Wigner formalism we see that it is, simply, a consequence of the fluctuating (in our case gaussian) character of the light beam of SPDC. The same result (72) is obtained for any classical chaotic light, like that of a lamp. We have shown elsewhere [32, 33] that it is also obtained when the source consists of a number of atoms, given that the times of emission cannot be controlled. The point is that there is no trace of corpuscular behaviour in the result (72). It is impossible to show any corpuscular behaviour of light by looking at radiation coming from a distant astronomical object, contrary to what has often been claimed.

The wave nature of light may be shown with the following experiment (see Fig.8), performed by Grangier et al.[23] with light coming from an atomic source, but here we shall analyze a similar experiment with a SPDC source. The experiment consists of recombining beams 2 and 3 of (66) at a second beam splitter. Then the outgoing beams, 4 and 5, will be given by

$$E_4^{(+)}(t) = \frac{1}{\sqrt{2}}\left(E_2^{(+)}(t)e^{i\phi} + iE_3^{(+)}(t)\right),$$
$$E_5^{(+)}(t) = \frac{1}{\sqrt{2}}\left(E_3^{(+)}(t) + iE_2^{(+)}(t)e^{i\phi}\right), \tag{73}$$

where the angle ϕ takes account of the difference in the optical path lengths, which may be varied by moving M1. By a calculation similar to the one leading to (72) it is straightforward to get

$$R_4 = \eta \mathcal{I}_1 \cos^2 \phi \quad , \quad R_5 = \eta \mathcal{I}_1 \sin^2 \phi \; , \tag{74}$$

which shows interference, a characteristic trait of waves. (In Fig.8 only one detector is shown.)

In order to have results similar to (64) and (65), showing the alleged corpuscular behaviour of light, it is necessary to perform a more sophisticated experiment. In it we must start with two beams having *entangled photons*, like the signal and idler produced in SPDC or the two beams of different colour produced by atomic cascades. With the latter source it was performed by Grangier et al. [23] (but see comment[53]). We shall analyze it with an SPDC source. There is a detector, D_2, in the signal beam, whilst the idler impinges on channel 1 of a beam splitter. There are two more detectors, D_3 and D_4, in the outgoing channels of the beam splitter (see Fig.9).

The measurable quantities are the singles rate R_2, the coincidence rates R_{23} and R_{24} and the triple coincidence rate R_{234}. In practice, when detector D_2 fires, detectors D_3 and D_4 are allowed to detect during some time window, τ, which should be of the order of the correlation time given by the function $\mu(t' - t)$ of (72). The triple coincidence is

$$R_{234} = \eta^3 \int_0^\tau dt' \int_0^\tau dt'' F \; , \tag{75}$$

where

$$F = \langle (I_2(t) - I_0)(I_3(t+t') - I_0)(I_4(t+t'') - I_0) \rangle \ . \tag{76}$$

Then, using the gaussian property (70), we get

$$\begin{aligned} F &= \langle I_2 - I_0 \rangle \langle I_3 - I_0 \rangle \langle I_4 - I_0 \rangle && (77) \\ &+ \langle I_2 - I_0 \rangle \left| \left\langle E_3^+ E_4^- \right\rangle \right|^2 c^2 \epsilon_0^2 && (78) \\ &+ \langle I_3 - I_0 \rangle \left| \left\langle E_2^+ E_4^+ \right\rangle \right|^2 c^2 \epsilon_0^2 && (79) \\ &+ \langle I_4 - I_0 \rangle \left| \left\langle E_2^+ E_3^+ \right\rangle \right|^2 c^2 \epsilon_0^2 \ , && (80) \end{aligned}$$

where we have suppressed the time dependence for simplicity. From the auto- and cross-correlation properties of beams produced in SPDC, we see that the first two terms are of order G^6 and the latter two of order G^4 so that R_{234} is of overall order G^4. In contrast R_2, R_{23} and R_{24} are all of order G^2. Consequently, as $G \ll 1$, we have

$$\alpha_G \equiv \frac{R_{234} R_2}{R_{23} R_{24}} \ll 1. \tag{81}$$

The dimensionless quantity α_G was taken as a substitute for the rather formal quantity (65) by Grangier et al.[23]. The small value found in their experiment has been interpreted as proof of the corpuscular nature of light. In our analysis, however, the experiment may be explained in terms of correlations between waves, together with a threshold in the detection process. It is the interference between the signal field E_3 and the ZPF input E_0 at BS which explains the anticorrelation counts in the outgoing channels. The interference may be constructive in channel 3 and destructive in channel 4 or viceversa, but it cannot be constructive in both. That is why a detection event occurs in only one of these channels. A similar analysis may be made of the earlier anticorrelation experiment of Clauser[22].

5.2 The Bell inequalities

In 1964 Bell[4] proved that there are predictions of quantum mechanics which cannot be reproduced by any local hidden variables (LHV) theory. It therefore may seem possible to discriminate between quantum mechanics and local realism (i.e. the whole family of LHV theories) by suitably designed experiments. Experiments attempting to do this, whether or not they are fully adequate, we shall call simply *Bell tests*. Actually most of such tests made during the last 20 years have used photon pairs produced in the process of SPDC [9, 10]. In general they have confirmed quantum mechanics but, in spite of great efforts, the fact is that they have been unable to disprove local realism. Indeed we shall prove that our approach provides a LHV model for all SPDC experiments, by combining the Wigner version of quantum optics, applied to the production and propagation of light, with the detection model we presented in section 3.

According to Bell [4], the crucial difference between quantum mechanics and LHV theories occurs in experiments where we measure the probabilities of the results of some measurements performed at space-like separation (EPR experiments[1]). In practice the measurements usually consist of detecting two particles after each one has passed through a two-channel device such as a polarizer or a Stern-Gerlach type apparatus. Any LHV model should contain hidden variables λ, with a probability distribution $\rho(\lambda)$, giving the following single and joint detection probabilities:

$$\begin{aligned} p_1 &= \int \rho(\lambda) P_1(\lambda, \phi_1) d\lambda \ , \\ p_2 &= \int \rho(\lambda) P_2(\lambda, \phi_2) d\lambda \ , \\ p_{12} &= \int \rho(\lambda) P_1(\lambda, \phi_1) P_2(\lambda, \phi_2) d\lambda \ , \end{aligned} \tag{82}$$

where ϕ_1 and ϕ_2 represent controllable parameters of the two-channel device, and $P_1(\lambda, \phi_1)$, $P_2(\lambda, \phi_2)$ are functions which give the detection probabilities for the two particles after emerging from the device. If no further restrictions are imposed, a model resting upon eqs.(82) is always possible, but in order to have a LHV model the functions P_1, P_2 , and $\rho d\lambda$ should be probabilities, and consequently the following conditions should also be fulfilled

$$\begin{aligned} &\rho(\lambda) \geq 0 \quad , \quad \int \rho(\lambda) d\lambda = 1 \ , \\ &0 \leq P_1(\lambda, \phi_1), P_2(\lambda, \phi_2) \leq 1 \ . \end{aligned} \tag{83}$$

The Wigner formalism of quantum optics provides an explicit LHV model for quantum optical experiments. We simply take the field amplitudes $\{\alpha_{\mathbf{k}}\}$ in place of the hidden variables λ and the Wigner function $\mathrm{W}(\{\alpha_{\mathbf{k}}\})$ in place of the function $\rho(\lambda)$. But in order to get an LHV model the Wigner function should be nonnegative and, in addition, we should introduce some nonnegative functions of the random variables $\{\alpha_{\mathbf{k}}\}$, which could play the role of the functions $P_i(\lambda, \phi_i)$ of eqs. (82) . The Wigner function is nonnegative, at least if the source of radiation is SPDC, as we showed in section 4. We shall show how to define functions $P_i(\lambda, \phi_i)$ in the next section.

5.3 Analysis of a Bell test using type-II parametric down conversion

Early Bell tests[5, 6, 7, 8] used atomic cascades. An analysis of this series of tests, based on a primitive version of the present theory, was made by us in [54]. Most Bell tests since 1982 have been made using SPDC in nonlinear crystals. Initially they used type-I parametric down-conversion in which the two correlated beams have the same polarization. In [34, 42, 43] experiments of this kind were analyzed in the Wigner function formalism. However, more recent experiments, using type-II phase matching, provide a more direct way to generate "entangled-photon" states. Type-II experiments are themselves of two types. In the first, that is collinear type-II SPDC[55], the crystal is oriented so that the ordinary and extraordinary radiation cones are mutually tangent in the direction of the pumping beam. To date, nearly all type-II experiments have used collinear phase matching. On the other hand [9], in noncollinear type-II phase matching, the two cones intersect along two directions, and this gives rise to an entangled state in the polarization (see Fig.10). It has been claimed that such a source produces true entangled states, capable of violating Bell's inequalities.

The beams 1 and 2 are selected and sent to two polarizers P_1 and P_2 oriented at angles ϕ_1 and ϕ_2 with respect to the polarization of the extraordinary ray. Coincidence rates were measured as functions of angles ϕ_1 and ϕ_2. In [9] additional optical devices, that is half- and quarter-wave plates, were used in order to produce four different Bell states, but we shall confine our analysis to just one of these states, namely the one which uses no additional devices.

Let us see how the polarization-entangled state is represented in the Wigner formalism. The two beams, coming out of the crystal along the directions where the ordinary and extraordinary cones intersect, are given by

$$\mathbf{E}_1^{(+)}(\mathbf{r},t) = E_e^{(+)}(\mathbf{r},t)\mathbf{i} + E_{o'}^{(+)}(\mathbf{r},t)\mathbf{j},$$
$$\mathbf{E}_2^{(+)}(\mathbf{r},t) = E_{e'}^{(+)}(\mathbf{r},t)\mathbf{i}' + E_o^{(+)}(\mathbf{r},t)\mathbf{j}', \tag{84}$$

where $\mathbf{i}, \mathbf{i}'$ represent the polarizations of the extraordinary beams and $\mathbf{j}$, $\mathbf{j}'$ the polarizations of the ordinary beams. The essential point is that the extraordinary component, $E_e^{(+)}$, of the first ray and the ordinary component, $E_o^{(+)}$, of the second ray are conjugated, and therefore correlated. Similarly, $E_{o'}^{(+)}$ and $E_{e'}^{(+)}$ are correlated, but $E_e^{(+)}$ ($E_o^{(+)}$) is uncorrelated to $E_{e'}^{(+)}$ ($E_{o'}^{(+)}$). Furthermore, the only nonzero correlations fulfil

$$\langle E_o^{(+)} E_e^{(+)} \rangle = \langle E_{o'}^{(+)} E_{e'}^{(+)} \rangle. \tag{85}$$

When a field $\mathbf{E}_1$ arrives at (an ideal) polarizer P_1 with polarization plane given by the unit vector $\mathbf{e}_1$, we should consider also the relevant ZPF incoming by the other channel. Then the outgoing fields are

$$\mathbf{E}_3^{(+)}(\mathbf{r}_1,t) = [\mathbf{E}_1^{(+)}(\mathbf{r}_1,t)\cdot\mathbf{e}_1]\mathbf{e}_1$$
$$+[\mathbf{E}_0^{(+)}(\mathbf{r}_1,t)\cdot\mathbf{e}_1']\mathbf{e}_1', \tag{86}$$

where $\mathbf{e}_1'$ is a unit vector perpendicular to $\mathbf{e}_1$. Here we recall that the action of a polarizer in classical optics is to project the electric field vector on the polarization direction, which in our approach is made in the first term of (86) for the signal and in the second for the ZPF entering the other channel.

In the same way we write for the field at the polarizer P_2 at time t'

$$\mathbf{E}_4^{(+)}(\mathbf{r}_2,t') = [\mathbf{E}_2^{(+)}(\mathbf{r}_2,t')\cdot\mathbf{e}_2]\mathbf{e}_2$$
$$+[\mathbf{E}_0'^{(+)}(\mathbf{r}_1,t')\cdot\mathbf{e}_2']\mathbf{e}_2', \tag{87}$$

where $\mathbf{E}_0'^{(+)}$ is the ZPF entering the second channel of the polarizer P_2.

The coincidence detection probability is

$$P_{34} = \eta^2 \int_t^{t+\tau} dt' \, \langle (I_3\,(t) - I_0)(I_4\,(t') - I_0) \rangle \, , \tag{88}$$

where

$$I_i(t) = \mathbf{E}_i^{(+)}(t)\cdot\mathbf{E}_i^{(-)}(t), i = 3, 4, \tag{89}$$

and, from now on, we remove the space dependence, which may be taken into account in a straightforward way [44]. Using the gaussian property (70) we obtain, from the latter two equations,

$$\begin{aligned} P_{34} &= \eta^2 \int_t^{t+\tau} dt' \{ \langle (I_3\,(t) - I_0) \rangle \langle (I_4\,(t') - I_0) \rangle \\ &\quad + c^2 \epsilon_0^2 \sum_{kl} \left| \langle E_{3k}^{(+)}(t) E_{4l}^{(+)}(t) \rangle \right|^2 \}, \end{aligned} \tag{90}$$

where (k, l) label the components of the vectors and we have ignored terms which are zero because the fields involved are not correlated. The first term of (90) is of order G^4 and therefore small with respect to the second one which is of order G^2. Retaining only the second term and taking account of the correlations mentioned in eq.(84) and the fact that the ZPF inputs $\mathbf{E}_0$ and $\mathbf{E}_0'$ are uncorrelated with the signals and with each other, we have

$$P_{34} = \eta^2 c^2 \epsilon_0^2 \int_t^{t+\tau} dt' \left| \langle E_o^{(+)}\,(t)\, E_e^{(+)}\,(t') \rangle \right|^2 \sin^2(\phi_1 - \phi_2) \tag{91}$$

where eq.(85) has been taken into account and we have defined

$$(\mathbf{i} \cdot \mathbf{e}_1) = \sin \phi_1 \quad , \quad (\mathbf{j} \cdot \mathbf{e}_2) = \sin \phi_2 .$$

The dependence of P_{34} on the angle $(\phi_1 - \phi_2)$ has been measured in the experiment [9] and corresponds to a 100% contrast. This type of correlation is usually claimed to violate a Bell inequality, but here we have shown that it is compatible with local realism, provided that the detection efficiency is low enough. If we wanted to calculate the coincidence probability with high efficiency we should use the detection theory outlined in section 3 and not just the linear approximation used here.

It is interesting to see how our approach gives an explanation for the violation of the no-enhancement hypothesis of Clauser and Horne [16], which states that

> A light signal's detection probability cannot increase as a result of it passing through a polarizer.

In the linear detection theory this means that the following inequality should hold for any realization of the incoming signal

$$\mathbf{E}_1^{(+)}(t) \cdot \mathbf{E}_1^{(-)}(t) \geq \langle \mathbf{E}_3^{(+)}(t) \cdot \mathbf{E}_3^{(-)}(t') \rangle_{\mathrm{ZPF}} \,. \tag{92}$$

Observe that no average is required on the left hand side, but an average over the ZPF input to the polarizer is required on the right hand side. Using eq.(86), we see that eq.(92) may be written

$$\begin{aligned}\mathbf{E}_1^{(+)}(t)\cdot\mathbf{E}_1^{(-)}(t) \geq \\ &\left|\left[\mathbf{E}_1^{(+)}(t)\cdot\mathbf{e}_1\right]\right|^2+\left|\left\langle\left[\mathbf{E}_0^{(+)}(t)\cdot\mathbf{e}_1\right]\right\rangle\right|^2 \\ &=\left|\left[\mathbf{E}_1^{(+)}(t)\cdot\mathbf{e}_1\right]\right|^2+\tfrac{1}{2}(c\epsilon_0)^{-1}I_0. \end{aligned} \tag{93}$$

The inequality is obviously violated if the signal is polarized so that the vector $\mathbf{E}_1^{(+)}$ is parallel to $\mathbf{e}_1$, which proves the violation of the no-enhancement hypothesis for some signals. However it may be seen that condition (92), averaged over all signals, is fulfilled.

6 Conclusion

We have shown that a wide class of quantum optical experiments may be interpreted in terms of a Maxwell, that is purely wave, theory of light by taking the Wigner function of the light field as a probability distribution. This requires that the Wigner function is nonnegative definite, which seems to be the case provided we do not make inappropriate idealizations of the experimental situations.

The idea behind such an interpretation of the Wigner function is that the electromagnetic quantum vacuum fluctuations, or zeropoint field, is a real field having the same nature as signals, but then the problem is to explain why photodetectors are not fired by the zeropoint alone. We solve it by proposing a model which contains a detection threshold, so that the zeropoint is effectively subtracted. Our model gives predictions which depart from the standard quantum ones for weak signals and/or high detection efficiency. We argue that these predictions do not contradict quantum theory, but only some idealizations of it, arising from approximations such as using first-order perturbation theory. Combined with the Wigner formalism it provides an explicit local hidden variables model for all Bell tests using parametric down conversion sources. Furthermore it gives hints for the domain where departures from standard quantum theory are to be expected. We have also predicted the new phenomenon of Spontaneous Parametric Up Conversion.

We show that the so called nonclassical effects of light are produced whenever the signal radiation is correlated with the noise (the zeropoint field). In particular this gives a clear picture of "photon entanglement", and explains how a wave theory can explain the anticorrelation produced by a beam splitter ("corpuscular behaviour of light"), as well as the enhancement of some light signals passing through a polarizer. The latter is a phenomenon demonstrated experimentally by early Bell tests.

Our description of the detection process has many features in common with that proposed by Gilbert, Oppy and Sulcs[56, 57], based on what those authors call *stochastic resonance*. However, they consider only the effect of a noise intervention at the point of detection, so that the description of nonclassical states, produced by the mixing of different field modes at a beam splitter or a nonlinear crystal, forms no part of their description. Indeed they propose an experiment[58] which would distinguish between their semiclassical theory and all theories which include entanglement. If such an experiment were to give what they predict, it would seriously challenge our theory as well as quantum optics.

To summarize, our definition of reality is in terms of states whose Wigner function is positive. States not falling within that category are no more than formal objects, which may nevertheless be of some use for computation. The situation is similar to what happens when we solve a classical diffusion equation[59] using a Fourier expansion, where the individual terms in the expansion are not positive definite but the sum, representing the density of the diffusing material, is always positive. In the optical case the resulting theory is semiclassical, rather than classical, because we have no interpretation for the quantum nature of material particles.

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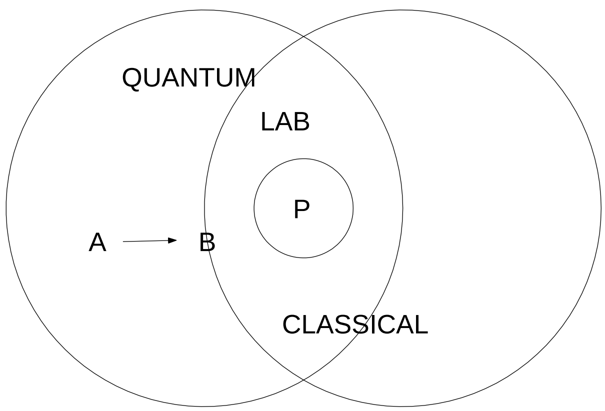

Figure 1: Classification of states of the radiation field. The restricted set of classical states whose Glauber P-distributions are nonsingular and positive is represented by the interior of the small circle.

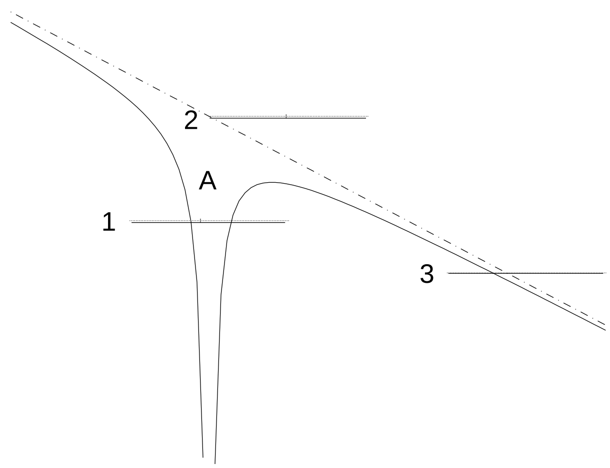

Figure 2: Schematic view of the potential seen by an electron in a detector. The broken line represents the potential created by an applied homogeneous electric field. A is the position of an atom. The three states mentioned in the text are represented by horizontal lines.

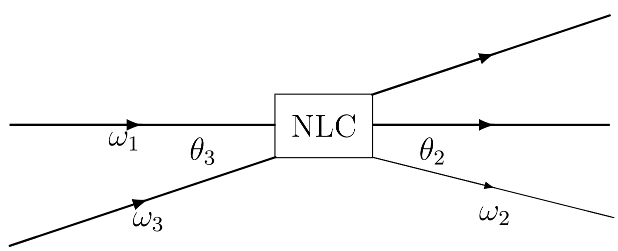

Figure 3: Parametric down conversion

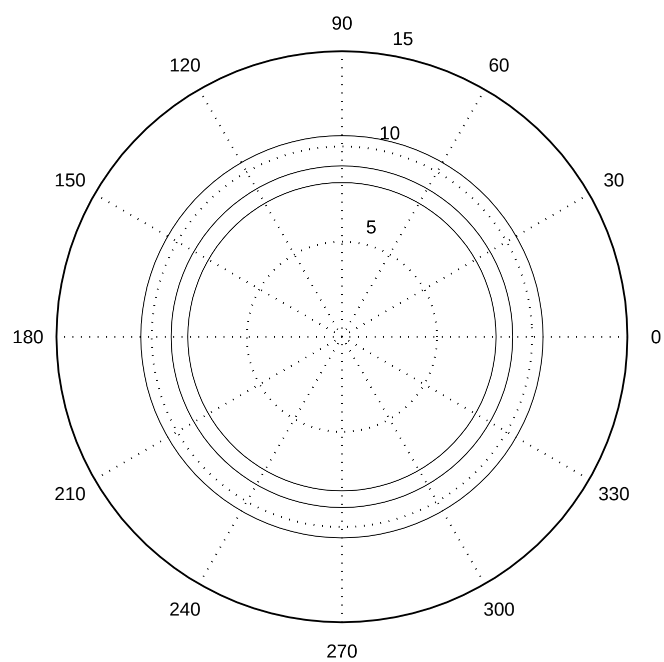

Figure 4: Position of the SPDC rainbow produced when a 351nm laser is normally incident on a BBO crystal cut with its axis at 37 degrees to the incident wave vector. The 600, 700 and 800nm components are shown; the 600 being the inner one and the 800 the outer one.

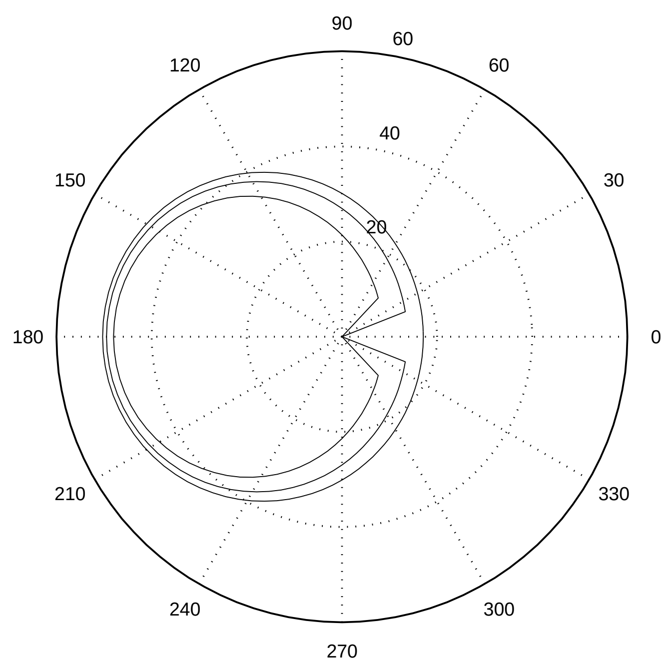

Figure 5: Position of the SPUC rainbow produced when a 845nm laser is normally incident on a BBO crystal cut with its axis at 37 degrees to the incident wave vector. The arcs of the 600, 700 and 800nm components are shown. Note that, in contrast with SPDC (Fig.4), the rainbow is not a complete circle, neither is it centred around the direction of the pumping laser.

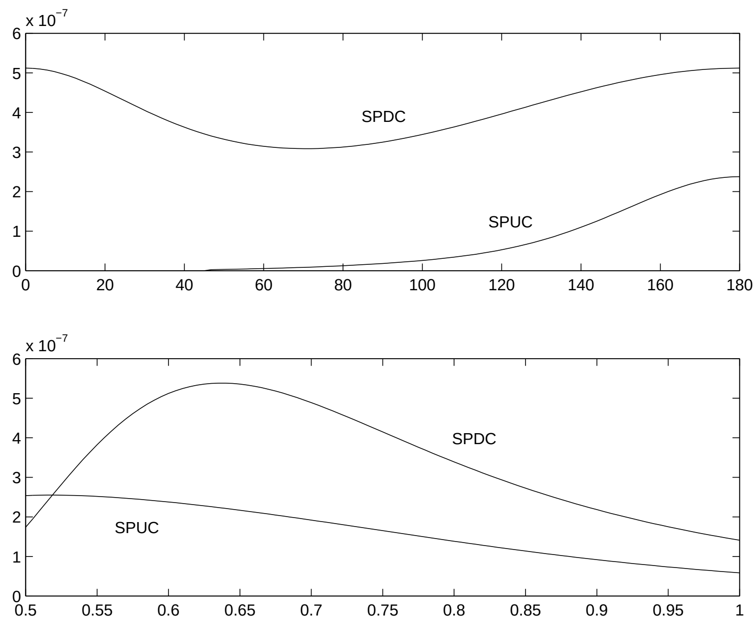

Figure 6: The SPUC and SPDC cross sections plotted against the azimuthal angle in degrees (upper figure, outgoing wavelength .6μm) and wavelength in μm (lower figure, outgoing azimuth 180 degrees) for a BBO crystal cut with its optic axis at 37 degrees to the normal of the incident face. The incident lasers have wavelengths .442 μm (SPDC) and .845 μm (SPUC)

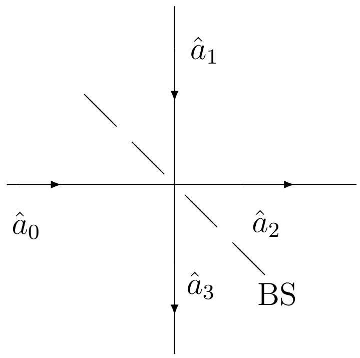

Figure 7: The action of a beam splitter

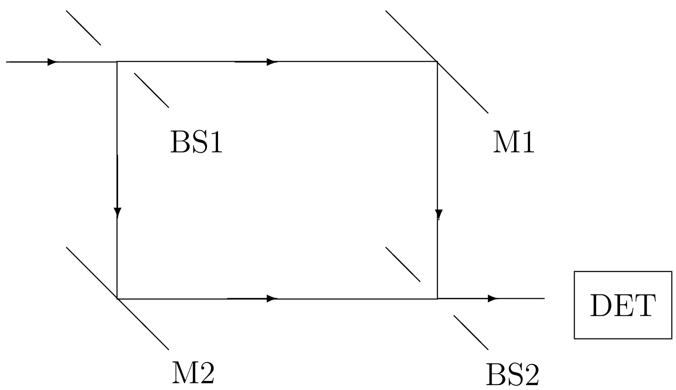

Figure 8: The recombination experiment. A beam is split at BS1 and recombined at BS2, after reflections at M1 and M2.

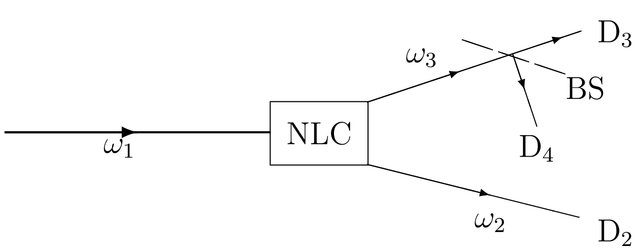

Figure 9: The anticorrelation experiment using SPDC. We count double coincidences between (D_2, D_3) and (D_2,D_4), and also triple coincidences

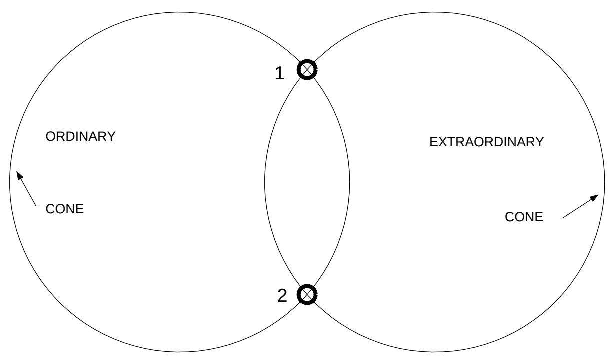

Figure 10: Polarization entanglement in noncollinear type II down conversion. The beams 1 and 2, in which the ordinary and extraordinary cones intersect, are analysed by polarizers P_1 and P_2.